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Non-linear analysis of piled rafts

Analyse non linéaire des fondations mixte radier-pieux

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ABSTRACT A practical analysis method for determining the response of piled rafts is described. The main feature of the method lies in its capability to provide a non-linear complete boundary element (BEM) solution of the soil continuum, while retaining a computationally efficient code. The negligible computational costs make the analysis suitable not only for the design of piled rafts supporting high rise buildings (generally based on complex and expensive 3D FEM or FDM analyses) but also for that of bridges and ordinary buildings.

RÉSUMÉ Cet article décrit une méthode d'analyse pour déterminer la réponse des fondations mixte radier-pieux. La principal caractéristique de la méthode réside dans sa capacité de fournir une solution non linéaire BEM du continuum sol, tout en conservant un code de calcul efficace. Le validité de l'analyse proposée est démontrée par comparaison avec d'autres solutions numériques. Les coûts négligeables de calcul rendent l'analyse appropriée non seulement pour la conception des radier-pieux supportant des immeubles de grande hauteur (basé sur complexes et coûteuses analyses 3D FEM ou FDM), mais aussi pour celle des ponts et bâtiments ordinaires.

1 INTRODUCTION

Piled rafts are a cost-effective foundation system which allows the load to be shared between the raft and the piles. In the design of piled rafts, a sufficient safety against geotechnical failure of the *overall* pile-raft system has to be achieved, while the piles may potentially be used up to their ultimate geotechnical capacity. Contrary to traditional pile foundation design, no proof for the ultimate capacity of each individual pile is necessary (Katzenbach & Choudhury 2013). Given the high load level at which the piles operate, consideration of soil nonlinearity effects is essential, and ignoring this aspect can lead to inaccurate predictions of the deformations and structural actions within the system.

Due to the 3D nature of the problem and the complexity of soil-structure interaction effects, calculation procedures for piled rafts are based on numerical analyses, ranging from Winkler approaches to rigorous 3D finite element (FEM) or finite difference (FDM) solutions using available packages. While Winkler models suffer from some restrictions mainly related to their semi-empirical nature and fundamental limitations (e.g. disregard of soil continuity), FEM and FDM solutions retain the essential aspects of interaction through the soil continuum, thereby providing a more realistic representation of the problem. However, even though 3D FEM and FDM analyses are powerful tools which allow complex geometries and soil behaviour to be modelled, such analyses are burdened by the high computational cost and specialist expertise needed for their execution, particularly if non-linear soil behaviour is to be considered. This aspect restricts their practical application in routine design, where multiple load cases need to be examined and where the pile number, properties and location may have to be altered several times in order to obtain an optimized solution. In an attempt to provide a practical tool for the designer, the paper presents an efficient numerical code based on the boundary element method (BEM) for computing the non-linear response of piled rafts. The validity of the proposed analysis is assessed through a comparison with a 3D FEM solution and a published case history.

2 ANALYSIS METHOD

The safe and economic design of piled rafts requires non-linear methods of analysis which have the capacity of simulating all relevant interactions between the foundation and the subsoil, specifically (1) pile-soil-interaction (i.e. single pile response including shaft-base interaction), (2) pile-pile-interaction (i.e. group effects), (3) raft-soil-interaction, and (4) pile-raft interaction, as illustrated in Fig. 1a (Katzenbach & Choudhury 2013).

The proposed analysis method is based on the BEM solution implemented in the piled-raft program PGROUPN (Basile 2015) and widely used in pile design through the software Repute (Bond & Basile 2012). The originality of the approach lies in its ability to provide a complete 3D BEM analysis of the soil continuum (in which all four of the above interactions are modelled), while incurring negligible computational costs. Indeed, compared to FEM or FDM analyses, BEM provides a complete problem solution in terms of boundary values only, specifically at the raft-pile-soil interface, thereby resulting in substantial savings in computing time and data preparation effort.

A detailed description of the theoretical formulation adopted in PGROUPN is reported in Basile (2015) and, hence, only a brief outline of the method is given below. Following the typical BEM scheme, the pile-soil interface is discretized into a number of cylindrical elements, while the raft-soil interface is discretized into rectangular (or triangular) elements, as shown in Fig. 1b. The analysis is based on a substructuring technique in which the piles, the raft, and the surrounding soil are modelled separately and then compatibility and equilibrium conditions at the raft-pile-soil interface are imposed. The soil is modelled using the well established Mindlin solution, the piles using the classical Bernoulli-Euler beam theory, while the raft is assumed to be fully rigid. Then, given unit boundary conditions, the pile, raft, and soil equations are combined together and solved, thereby leading to the distribution of displacements, stress, forces, and moments in the piled raft under the prescribed external loads. General loading conditions (vertical, horizontal, and moment loading) can be applied to the piled raft. However, while the vertical and moment load is carried partly by the piles and partly by the raft contact pressures, the lateral load is entirely taken by the piles, given that only the bearing contribution of the raft underside is considered (i.e. the raft-soil interface is assumed to be smooth).

Non-linear soil response is modelled, in an approximate manner, by adopting the common Duncan-Chang hyperbolic stress-strain model within a stepwise incremental procedure. The external loads on

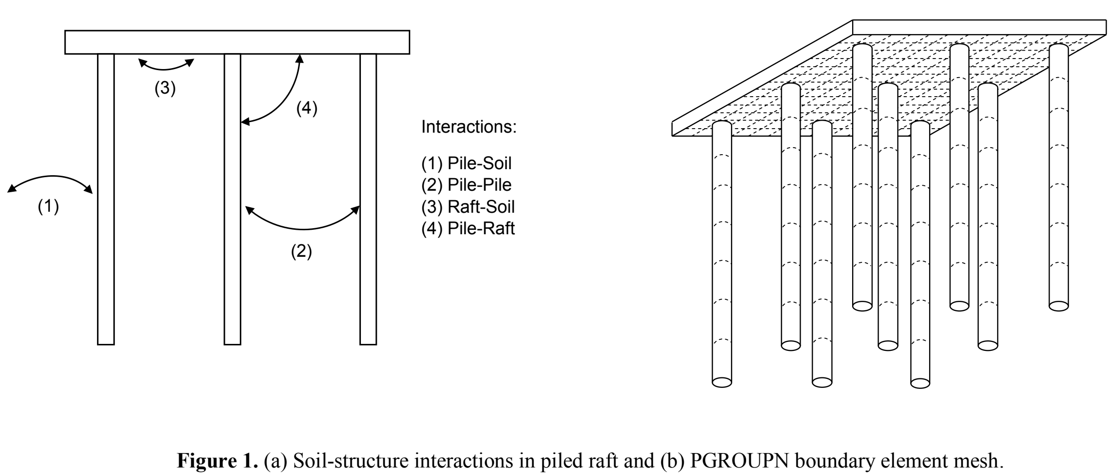


Figure 1. (a) Soil-structure interactions in piled raft and (b) PGROUPN boundary element mesh.

the piled raft are applied incrementally and, at each increment, a check is made that the stress state at the raft-pile-soil interface does not violate the yield criteria. This is achieved by specifying limiting values at the pile-soil interface according to the classical equations for the axial and lateral pile shaft capacity, and end-bearing resistance (Basile 2003). Similarly, limiting values of raft-soil contact pressure (based on the traditional bearing capacity theory) are set for both compression and tension in order to allow for local bearing failure or lift-off of the raft from the soil (Basile 2015). The elements of the raft-pile-soil interface which have yielded can take no additional load and any increase in load is therefore redistributed between the remaining elastic elements until all elements have failed. Thus, by successive application of loading increments, the entire load-displacement relationship for the piled raft is determined.

The PGROUPN analysis is currently restricted to the assumption of perfectly rigid raft. In practice, this assumption makes the analysis strictly applicable to "small" piled rafts (Viggiani et al. 2012), i.e. those rafts in which the bearing capacity of the unpiled raft is generally not sufficient to carry the applied load with a suitable safety margin, and hence the primary reason for adding piles is to increase the factor of safety. This typically involves rafts in which the width (B_r) amounts to a few meters (typically 5m < B_r < 15m) and is small in comparison to the length (L) of the piles (i.e. $B_r/L < 1$). Within this range (whose limits should however be regarded as tentative and indicative only), the raft response may be considered as truly rigid and hence the design should aim at limiting the maximum settlement (being the differential settlements negligible). In practical applications, a simple check on the validity of the assumption of rigid raft may be performed by calculating the raft-soil stiffness ratio (K_{rs}) as defined by Horikoshi & Randolph (1997):

$$K_{rs} = 5.57 \frac{E_r}{E_s} \frac{1-v_s^2}{1-v_r^2} \left(\frac{B_r}{L_r}\right)^{0.5} \left(\frac{t_r}{L_r}\right)^3 \qquad (1)$$

where the subscripts r and s denote the raft and soil properties, respectively, E is the Young's modulus, v is the Poisson's ratio, B_r is the raft width, L_r is the raft length (with $B_r \leq L_r$), and t_r is the raft thickness. For values of K_{rs} > 5-10, the raft can be considered as rigid while a lower limit K_{rs} > 1.5 may be assumed for practical purposes. It is however observed that the above definition of K_{rs} does not include the additional stiffening contribution provided by the piles and by the superstructure which in effect increases the actual raft rigidity. Clearly, for "large" flexible rafts (in which typically $B_r/L > 1$ according to the definition by Viggiani and colleagues), the assumption of rigid raft is no longer valid and the limitation of differential settlement becomes one of the design requirements. However, except for thin rafts, the maximum settlement and the load sharing between the raft and the piles are little affected by the raft rigidity.

3 NUMERICAL RESULTS

The validity of the proposed analysis is verified through a comparison with a 3D FEM solution and a published case history.

3.1 Comparison with Lee et al. (2010)

The behaviour of a square raft supported by 3x3 piles and embedded into a homogeneous soil layer is investigated. Two different pile spacings (s) are examined, i.e. s=3D and s=9D (where D is the pile diameter), as shown in Fig. 2. The piles are taken to be 0.5 m in diameter and 16 m in length, while the square raft has a width (B) of 10 m and a thickness of 1 m. The raft may be considered as fully rigid being K_{rs} = 31.7 from Equ. (1). The piled raft is embedded into a soft clay layer underlain by a rigid layer at a depth of 20 m. Attention is focused on the drained (long-term) response of the piled raft resting on the soft clay layer, so that the clay is modelled as a Mohr-Coulomb material using drained shear strength parameters, i.e. an effective cohesion (c') of 3 kPa and a friction angle (ϕ') of 20°. The clay has a drained Young's modulus (E_s) of 5 MPa and a Poisson's ratio (ν_s) of 0.3, with the ground water table located on top of the layer and assuming a hydrostatic water pressure distribution. The material parameters adopted in the analysis are summarised in Fig. 2.

In Figs. 2–4, results from PGROUPN are compared with those reported by Lee et al. (2010) using the 3D FEM software ABAQUS (2010). In order to describe the pile-soil interface behaviour, ABAQUS adopts a slip model using 2D quadratic 18-node ele

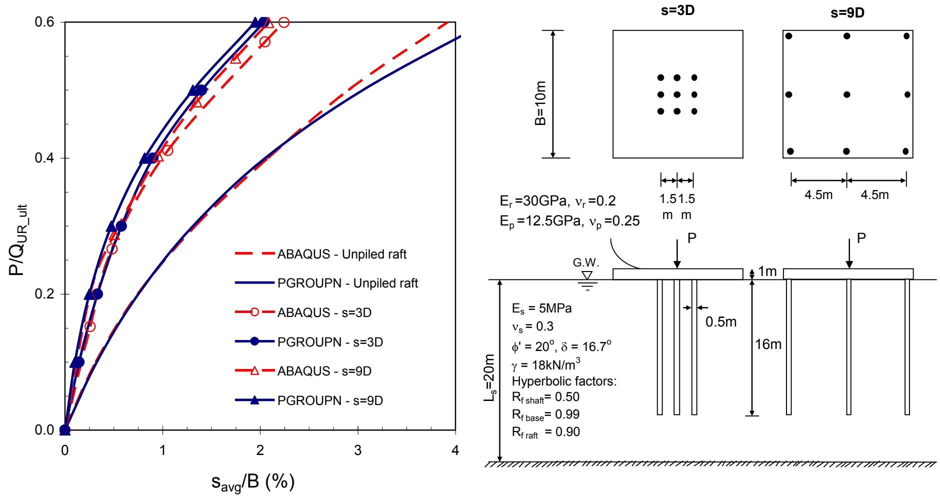


Figure 2. Normalized load-settlement response and piled rafts analysed.

ments with a friction coefficient of 0.3 (corresponding to an interface friction angle δ of 16.7°). Figure 2 shows the normalized load P/Q_{UR_ult} versus the average settlement s_{avg}/B of the piled rafts. In addition, the response of the unpiled raft is reported. The ultimate bearing capacity of the unpiled raft (Q_{UR_ult}) used to normalize the applied load level (P/Q_{UR_ult}) can be estimated as equal to 25 MN, which corresponds to the load causing a settlement of 10%B in the ABAQUS analysis of the unpiled raft. As expected, the settlement increases with increasing load level and with decreasing pile spacing (due to the increase of pile-to-pile interaction effects).

The normalized pile load distribution along the corner and the centre pile of the piled raft is reported in Figs. 3a (s=3D) and 3b (s=9D). In these figures, Q_{P_PR} is the load taken by the pile beneath the piled raft, z is the depth from ground level, L_s is the depth to the rigid layer, and the ultimate bearing capacity of single pile (Q_{SP_ult}) can be estimated as equal to 522 kN. The above value corresponds to the load causing a settlement of 10%D in the corresponding single-pile ABAQUS analysis.

Figure 3c shows the proportion of load taken by the piles in the piled raft ($\alpha_{pr} = \Sigma Q_{P\_PR}/P$, where ΣQ_{P_PR} is the sum of all pile head loads and P is the

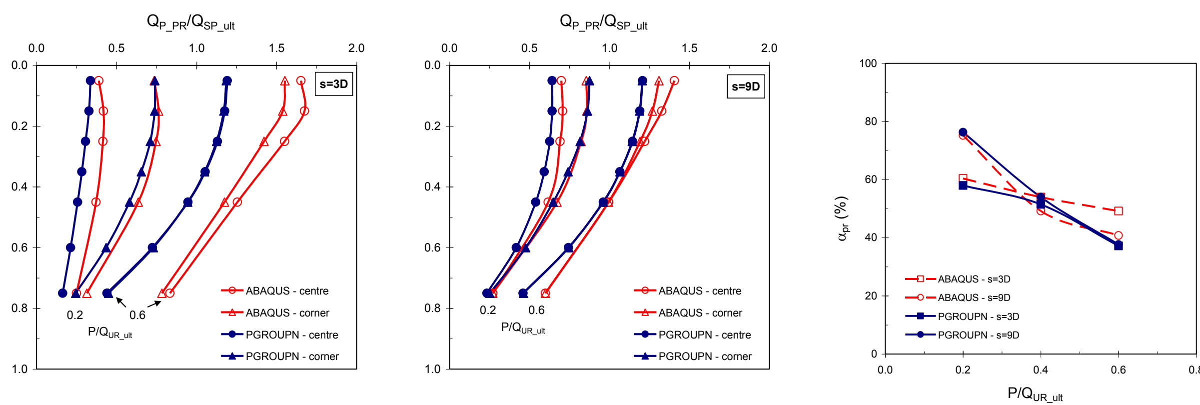


Figure 3. (a) Pile load profile in piled raft with s=3D, (b) Pile load profile in piled raft with s=9D, and (c) Piled raft coefficient.

applied total load) as a function of the applied load level. As expected, the load carried by the piles reduces with increasing load level due to the increasing raft's contribution as yielding along piles progresses.

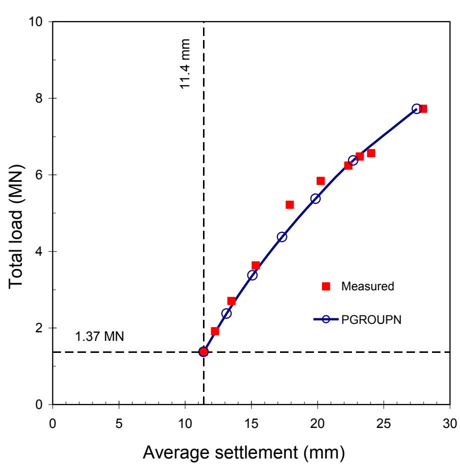
Overall, the comparison presented in Figs. 2–3 shows a favourable agreement between PGROUPN and ABAQUS, except for the case of closely spaced piles (s=3D) at high load levels. This difference is due to the higher bearing capacity of the pile group calculated by ABAQUS in the case of closely spaced piles. As a consequence, at an applied load level (P/Q_{UR_ult}) of 0.6, ABAQUS predicts a higher proportion of load taken by the piles in the case of narrow pile spacing (Fig. 3a) as compared to the case of wide pile spacing (Fig. 3b). Instead, in PGROUPN, the ultimate bearing capacity of the pile group is mobilized at an identical load level (P/Q_{UR_ult} = 0.6) for both the narrow and the wide pile spacing.

The comparison also demonstrates the importance of considering soil nonlinearity effects in order to obtain realistic predictions of the settlement and the load sharing between the raft and the piles. Indeed, assumption of linear elastic behaviour beyond a load level (P/Q_{UR_ult}) of 0.2 would lead to an underestimation of the settlement (Fig. 2) and an overestimation of the amount of load carried by the piles (Fig. 3c), with a consequent over-design of the requirements for structural strength of the piles.

Finally, it is noted that the ABAQUS analysis adopts a large and time-consuming 3D mesh including 27-node 2nd order hexahedral elements. For comparison, the mesh required by PGROUPN (following the scheme depicted in Fig. 1b) includes 153 pile cylindrical elements and 432 raft rectangular elements, resulting in a computational time of only 2 min on an ordinary computer (Intel Core i7 2.7 GHz).

3.2 Comparison with Kakurai et al. (1987)

Kakurai et al. (1987) described a full-scale loading test on a piled raft in soft cohesive soil, as illustrated in Fig. 4. The structure is a reinforced concrete silo that contains coal with a total weight (including the silo, the foundation, and the stock material) of 7.72 MN. The five piles are closed-end steel pipe piles with an embedded length of 22.7 m, an external diameter of 400 mm, a wall thickness of 9 mm, and a Young's modulus of 200 GPa. The piles are capped by a 0.6 m thick reinforced concrete raft. The soil consists of a soft alluvium stratum from ground level to a depth of 44 m where a stiff sandy layer is located. The upper silty layer at the depth of 2 to 5 m is overconsolidated with an average SPT value N = 5. The soil modulus (E_s) obtained by a self-boring pressuremeter (SBP) test under undrained condition shows a nearly constant value of 16 MPa down to a depth of 14.6 m, then tends to increase linearly to a value of 32.8 MPa at the pile base level, and then increases further to a value of 103.9 MPa at a depth of 32.3 m below ground level. A constant value of 0.5 for the Poisson's ratio is assumed throughout the soil.



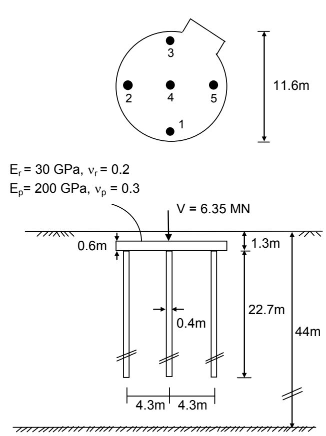


Figure 4. Load-settlement response and piled raft analysed.

Table 1. Soil parameters used in PGROUPN analysis.

Layer depth (m)	E_s (MPa)	m_{Es} (MPa/m)	C_u (kPa)	m_{Cu} (kPa/m)	α
0.0	20.0	0.0	20.0	0.0	1.0
5.0	16.0	0.0	20.0	0.0	1.0
9.0	16.0	0.0	20.0	5.5	0.89
14.6	16.0	1.83	51.0	5.5	0.5
23.8	32.8	8.36	102.0	5.5	0.5

The undrained shear strength (C_u) shows a nearly constant value of about 20 kPa down to a depth of 9 m, and then tends to increase linearly at a rate of 5.5 kPa/m. Based on the above soil properties, the soil parameters adopted in PGROUPN are shown in Table 1. It is observed that the comparison of response presented herein refers to short-term undrained conditions, i.e. the measurements are considered at the time of 'end of construction' when the silo is first filled with the stock material. It is also noted that K_{rs} = 1.3 results from Equ. (1) and, hence, the PGROUPN assumption of rigid raft can reasonably be applied, as confirmed by the actual field measurements.

The load-settlement response of the piled raft is reported in Fig. 4 and shows a good agreement between numerical results and field measurements. It is noted that an immediate settlement of 11.4 mm was observed under a total load of 1.37 MN because the most load was directly transferred to the soil during the concrete casting of the raft. Thus, the PGROUPN analysis has been conducted under an actual total load of 6.35 MN (i.e. 7.72-1.37 MN) which corresponds to the load increase after the piled-raft system starts operating. The settlement reported in Fig. 4 represents the average settlement among piles; due to the asymmetrical shape of the raft, both the measured settlements and pile top loads were greater at one side (piles 1 and 2) and less at the other side (piles 3 and 5). This feature of behaviour is confirmed by PGROUPN which calculates the largest settlement on pile 1 (w = 28.4 mm) and the smallest one on pile 3 (w = 26.5 mm) due to tilting of the raft.

It is observed that the raft took a significant proportion of the total applied load with a favourable agreement between measured (62%) and calculated (55%) value. Thus, the piled raft foundation has proved to be a cost-effective solution as compared to a traditional pile-group solution in which the raft contribution is ignored.

4 CONCLUSIONS

The paper has illustrated a practical approach, based on a 3D complete BEM analysis and implemented in the code PGROUPN, for determining the non-linear response of piled rafts. It has been shown that the concept of piled raft, generally adopted for "large" flexible piled rafts, can also be applied effectively to "small" rigid piled rafts (and to any larger piled raft in which the assumption of rigid raft is valid), making PGROUPN suitable to a wide range of foundations such as bridges, viaducts, wind turbines, and ordinary buildings (where use of 3D FEM or FDM analyses would be uneconomical). In such cases, if the raft can be founded in competent ground, then the extra raft component of capacity can be used to significantly reduce the piling requirements which are necessary to achieve the design criteria.

Given the relatively high load level at which the piles operate within a pile-raft system, the influence of soil nonlinearity can be significant, and ignoring this aspect can lead to inaccurate predictions of the deformations and the load sharing between the raft and the piles. Due to the negligible costs (both in terms of data preparation and computer execution times), a large number of cases can be analysed efficiently, enabling parametric studies to be readily performed, thus offering the prospect of more effective design techniques and worthwhile savings in construction costs.

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